$$J_{\varphi \bar{f}}(\bar{x}) = \varphi(\bar{x}) \cdot J_{\bar{f}}(\bar{x}) + \nabla\varphi(\bar{x}) \cdot \bar{f}(\bar{x}) \quad \text{οπου} \quad \bar{f} = \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$$

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$$\varphi\bar{f} = \begin{pmatrix} \varphi f_1 \\ \vdots \\ \varphi f_m \end{pmatrix}$$

$$J_{\varphi \bar{f}}(\bar{x}) = \begin{pmatrix} \frac{\partial \varphi f_1}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial \varphi f_1}{\partial x_n}(\bar{x}) \\ \vdots & & \vdots \\ \frac{\partial \varphi f_m(\bar{x})}{\partial x_n}(\bar{x}) & \cdots & \frac{\partial \varphi f_m}{\partial x_n}(\bar{x}) \end{pmatrix} =$$

$$= \begin{pmatrix} \varphi(\bar{x}) \cdot \frac{\partial f_1}{\partial x_1}(\bar{x}) + \frac{\partial \varphi}{\partial x_1}(\bar{x}) \cdot f_1(\bar{x}) & \cdots & \varphi(\bar{x}) \frac{\partial f_1}{\partial x_n}(\bar{x}) + \frac{\partial \varphi}{\partial x_n}(\bar{x}) \cdot f_1(\bar{x}) \\ \vdots & & \vdots \\ \varphi(\bar{x}) \cdot \frac{\partial f_m}{\partial x_1}(\bar{x}) + \frac{\partial \varphi}{\partial x_1}(\bar{x}) \cdot f_m(\bar{x}) & \cdots & \varphi(\bar{x}) \cdot \frac{\partial f_m}{\partial x_n}(\bar{x}) + \frac{\partial \varphi}{\partial x_n}(\bar{x}) \cdot f_m(\bar{x}) \end{pmatrix} =$$

$$= \begin{pmatrix} \varphi(\bar{x}) \frac{\partial f_1}{\partial x_1}(\bar{x}) & \cdots & \varphi(\bar{x}) \frac{\partial f_1}{\partial x_n}(\bar{x}) \\ \vdots & & \vdots \\ \varphi(\bar{x}) \frac{\partial f_m}{\partial x_1}(\bar{x}) & \cdots & \varphi(\bar{x}) \frac{\partial f_m}{\partial x_n}(\bar{x}) \end{pmatrix} + \begin{pmatrix} f_1(\bar{x}) \frac{\partial \varphi}{\partial x_1}(\bar{x}) & \cdots & f_1(\bar{x}) \frac{\partial \varphi}{\partial x_n}(\bar{x}) \\ \vdots & & \vdots \\ f_m(\bar{x}) \frac{\partial \varphi}{\partial x_1}(\bar{x}) & \cdots & f_m(\bar{x}) \frac{\partial \varphi}{\partial x_n}(\bar{x}) \end{pmatrix} =$$

$$= \varphi(\bar{x}) \cdot \begin{pmatrix} \frac{\partial f_1(\bar{x})}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}(\bar{x}) \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial f_m}{\partial x_m}(\bar{x}) \end{pmatrix} + \nabla\varphi(\bar{x}) \cdot \begin{pmatrix} f_1(\bar{x}) & \cdots & f_1(\bar{x}) \\ \vdots & & \\ f_m(\bar{x}) & \cdots & f_m(\bar{x}) \end{pmatrix} =$$

$$= \varphi(\bar{x}) \cdot J_{\bar{f}}(\bar{x}) + \nabla\varphi(\bar{x}) \cdot \bar{f}(\bar{x}) \cdot \in \mathbb{R}^{n \times m}.$$